A DISCUSSION ON FIRST LAW OF BLACK HOLE MECHANICS USING GRAVITATIONAL WAVE

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Abstract

The discovery of gravitational wave gives a new way to look at the laws of the black hole mechanics. There are four laws of black hole mechanics. Like the first law of thermodynamics, the first law of black hole mechanics is essential for comprehending the dynamics of black holes. Recent developments in gravitational wave astronomy have opened a special way to study black hole mechanics. The second law of black hole mechanics is fully explored using the gravitational wave data but the first law is not fully tested yet. In this article, we discuss the first law of black hole mechanics using gravitational wave briefly.

1. Introduction

Newton's theory states that space and time are absolute quantities, and the interaction between two objects occurs instantaneously. However, according to Einstein's special theory of relativity, space and time are not absolute concepts and no information can travel faster than the speed of light. According to general theory of relativity, gravity is not instantaneous rather the curvature of the space-time fabric [1]. Gravitational waves are ripples in the spacetime curvature that propagate outward from a source at almost the speed of light [2]. Gravitational waves would be produced by any cosmic event that caused enough force to alter the fabric of space time.

In 1915, Einstein formulated the general theory of relativity. Gravitational waves were initially predicted by Einstein theoretically in 1916. Einstein's equations demonstrated that massively speeding objects (such as neutron stars or black holes orbiting each other) would disturb space-time, causing waves of undulating spacetime to travel in all directions away from the source. Any object having mass and moving with acceleration produces gravitational waves. But the gravitational waves produced by the objects on the earth are too small to detect by the current technology. Generally heavy (massive and compact) astronomical objects, such as binary systems like neutron stars and black holes emit gravitational waves that are currently detectable.

The first experimental observation of gravitational wave signal by the coalescence of two black holes was made by Laser Interferometer Gravitational Wave Observatory (LIGO) detectors at Livingston and Hanford on 14th September, 2015 which ensured the physical existence of black holes and binary black hole system. The LIGO has identified four types of gravitational waves on the basis of their sources till now [3]. They are:

Continuous Gravitational Wave: This form of gravitational wave is generated by solitary rotating large objects such as neutron stars and black holes. This form of gravitational wave has constant frequency and amplitude of radiation.

Compact Binary Inspiral Gravitational Wave: Each binary pair generates a distinct series of gravitational waves but the mechanism for wave formation is the same; it is known as inspiral. These waves are typically short (one to several seconds) and increase in frequency as they orbit faster.

Stochastic Gravitational Wave: A stochastic gravitational wave is formed by the random mixing of many small gravitational waves from all throughout the universe.

Burst Gravitational Wave: Burst gravitational waves come from short duration unknown sources. There is hypothesis that some systems as supernova or gamma ray burst may produce burst gravitational waves.

Gravitational waves are significant because they offer a new means of observing and comprehending cosmic events such as black hole mergers or collision of neutron stars. They also provide a unique means of verifying and testing Einstein's theory of general relativity. Finally, gravitational waves address fundamental questions in physics such as the nature of gravity itself and possible new physics beyond the standard model. In summary, gravitational waves enhance our comprehension of the universe by providing a novel means of observation and by strengthening our grasps of the fundamental laws governing cosmic phenomena.

The discovery of gravitational wave gives a new way to look at the laws of the black hole mechanics. There are four laws of black hole mechanics. The second law of black hole mechanics is fully explored using the gravitational wave data but the first law is not fully tested yet. In this article, we discuss the first law of black hole mechanics using gravitational wave briefly.

2. Physical properties of gravitational wave

Gravitational waves (GWs) are transverse in nature and move at the speed of light. Basically, gravitational waves are oscillations in gravitational field. Generation of gravitational waves are similar with the generation of electromagnetic (EM) waves. The EM waves are generated due to oscillation of electric charges i.e. microscopic motion. On the other hand, GWs are generated due to macroscopic motion of any dynamical system. Gravitational waves have many similar properties with sound waves. Gravitational waves propagate through most of the materials like sound waves and weakly interact with matter. As gravitational waves propagate, they exert a periodic expansion and contraction of the space-time in direction perpendicular to the direction of travel. This property is used to detect the gravitational waves. Gravitational waves carry energy, momentum and angular momentum. Gravitational waves, like electromagnetic waves, vary wavelength and frequency due to relative velocities (Doppler Effect) and space-time distortions (e.g., cosmic expansion). Gravitational wave redshift differs from gravity induced redshift. The EM waves attenuate in the medium as these waves get absorbed and scattered but as GWs are weakly interacting with matter, these remain unaltered. Due to this property, these waves can penetrate regions which are not accessible to EM waves such as dynamics in violent phenomenon like supernovae explosion, binary star mergers, black hole dynamics etc. Due to this very weak interaction, it is very difficult to detect GWs.

3. Laws of black hole mechanics

One of the greatest predictions of general theory of relativity is black hole. Any object whose mass is concentrated into its Schwarzschild radius $\left(R_S = \frac{2GM}{C^2}\right)$ [4] can become a black hole. When a dead star collapses and passes the Schwarzschild radius it results in forming black hole. These giant objects in the cosmos distorts spacetime fabric and the warping of spacetime is so great that even light cannot escape. And these black holes become a perfect ground to test our understanding of the general theory of relativity and quantum gravity.

"Laws of black hole mechanics" refers to a series of comparisons made between the event horizon of black holes and the principle of thermodynamics. Physicist Jacob Bekenstein first put forward these comparisons [5] in the early 1970s and Stephen Hawking expanded on them. The surface area and surface gravity of black hole are comparable to the temperature and entropy of a thermodynamic system in black hole mechanics respectively. The acceleration of a test particle positioned at the event horizon equals the surface gravity (k) of black hole. It is given as $k = \frac{GM}{R_S^2}$ where R_S is the Schwarzschild radius. And area of event horizon is defined as the surface area of black hole ($A = 4\pi R_S^2$). The laws are as follows [6,7]:

Zeroth law of black hole mechanics: According to this law, a black hole's surface gravity (k) remains constant throughout its event horizon. This law suggests that black holes in equilibrium have constant surface gravity in resemblance to the zeroth law of thermodynamics.

First law of black hole mechanics: Similar to the first law of thermodynamics, this law asserts that the change in mass (dM) of a black hole is connected to the change in its area (dA), angular momentum (dJ), and electric charge (dQ) by the equation [8]:

$$dM = \frac{k}{8\pi} dA + \Omega dJ + V dQ$$

Here, κ is the surface gravity, Ω is the angular velocity of the horizon and V is the electrostatic potential.

Second law of black hole mechanics: This law states that no physical action can cause the area of a black hole's event horizon to shrink over time, drawing a connection with the second law of thermodynamics. In terms of math, this is stated as $\Delta A \geq 0$ [9] where ΔA is the change in the area of the event horizon.

Third law of black hole mechanics: Similar to the third law of thermodynamics, which says that as a system's temperature gets closer to zero, its entropy will approach a constant value. It would seem that a black hole with zero area cannot be created, according to the third law of black hole mechanics.

4. Testing the first law of black hole mechanics

A special method for investigating black holes and testing the rules governing them is through gravitational waves. Gravitational waves are released by merging black holes or other major events, and these waves carry information about the attributes of the black holes including mass, spin and angular momentum. Scientists may test theoretical hypothesis such as the rules of black hole mechanics and learn more about the behaviour of black holes by analysing the gravitational wave signals detected by the VIRGO and LIGO collaborations.

Due to their orbital decay, merging black holes release gravitational waves [10]. These mergers can be examined using the principles of black hole mechanics, especially the first law of, which specifies change in mass, angular momentum and charge. The laws of black hole mechanics and general relativity have been validated by gravitational wave observations of black hole mergers.

The analysis of these mergers involves the application of first law of black hole mechanics. We can verify whether the first law holds true by examining the characteristics of the ultimate black hole and the gravitational waves released during the merger. We can check whether the first law's prescribed relationship is followed when the mass, angular momentum and other features of the merging black holes change or not.

Using the gravitational wave data, the black hole area law, also called the second law of black hole mechanics and the BH-no hair theorem have been tested [11].

5. Methodology, Calculation and Results

The first law of black hole mechanics, also known as the Bekenstein-Smarr formula is given by

$$dM = \frac{k}{8\pi} dA + \Omega dJ + V dQ$$

where M, k, A, Ω, J, V, Q are the black hole mass, surface gravity, area of the event horizon, angular velocity, angular momentum, electronic potential and electronic charge of the black holes. This is analogous to the first law of thermodynamics $dE = TdS - PdV$. The first term represents the change in the energy of the black hole due to change in its area. The second and third term represents the work done on the black hole by the change in its angular momentum and charge respectively.

According to the first law, disturbances can cause an asymptotically flat stationary black hole spacetime to evolve into a new stationary black hole spacetime. As a result, an appropriate test of the first law of black hole necessitates a black hole with little disruption. In order to perceive the merging of a binary system as a perturbed process and test the first law of black holes, the gravitational wave chosen must have an exceptionally high mass ratio. However, the current gravitational wave events identified thus far cannot cause a minor disturbance.

We can rewrite the first law of black hole equation as [8]

$$\Delta M = (1-\alpha)\left(\frac{k}{8\pi}\Delta A + \Omega\Delta J\right)$$

This is also called the weak version of the first law of black hole mechanics where the symbol Δ represents the change of the physical quantity and α represents the deviation from the first law of black hole. We can neglect the charge term as most of the astronomical black holes are charge free. Now k, A, **Ω**, J are expressed in terms of the mass and dimensional spin magnitude χ as [8, 12]

$$k = \frac{\sqrt{1-\chi^2}}{M\left(1+\sqrt{1-\chi^2}\right)}$$

$$A = 8\pi M^2\left(1+\sqrt{1-\chi^2}\right)$$

$$\Omega = \frac{\chi}{2M\left(1+\sqrt{1-\chi^2}\right)}$$

$$J = \chi M^2$$

Using all of these parameters in the weak version of the first law of black hole mechanics, we can roughly test how much the first law of black hole mechanics is valid by lucid mathematical calculation. For this, collision of binary systems is a perfect tool to test the first of black hole mechanics.

(a) Using GW151226 [13]:

For Primary black hole, M =14.2, χ = 0.4

$$k = \frac{\sqrt{1-0.4^2}}{2\times 14.2\times\left(1+\sqrt{1-0.4^2}\right)} = 0.016872$$

$$A = 8\times 3.14\times 14.2^2\times\left(1+\sqrt{1-0.4^2}\right) = 9725.177$$

$$\Omega = \frac{0.4}{2\times 14.2\times\left(1+\sqrt{1-0.4^2}\right)} = 0.007335$$

$$J = 0.4\times 14.2^2 = 80.656$$

For final black hole, M =20.8, χ = 0.74

$$A = 8\times 3.14\times 20.8^2\times\left(1+\sqrt{1-0.74^2}\right) = 17606.025$$

$$J = 0.74\times 20.8^2 = 320.1536$$

Now
$\Delta M = 6.6,\ \Delta A = 7880.848,\ \Delta J = 239.5036$

From the first law of black hole mechanics

$$6.6 = (1-\alpha)\left[\frac{0.016872}{8 \times 3.14} \times 7880.848 + 0.007335 \times 239.5036\right]$$

$$1-\alpha = 0.93 \qquad \Rightarrow \qquad \alpha = 0.07$$

(b) Using GW190412 [14]

For Primary black hole, M =30.1, $\chi = 0.44$

$$k = \frac{\sqrt{1-0.44^2}}{2 \times 30.1 \times \left(1+\sqrt{1-0.44^2}\right)} = 0.00782$$

$$A = 8 \times 3.14 \times 30.1^2 \times \left(1+\sqrt{1-0.44^2}\right) = 43014.455$$

$$\Omega = \frac{0.44}{2 \times 30.1 \times \left(1+\sqrt{1-0.44^2}\right)} = 0.003867$$

$$J = 0.44 \times 30.1^2 = 398.64$$

For final black hole, M =37.3, $\chi = 0.67$

$$A = 8 \times 3.14 \times 37.3^2 \times \left(1+\sqrt{1-0.67^2}\right) = 60811.616$$

$$J = 0.67 \times 37.3 = 932.164$$

Now

$\Delta M = 7.2$, $\Delta A = 17797.161$, $\Delta J = 533$

From the first law of black hole mechanics

$$7.2 = (1-\alpha)\left[\frac{0.00782}{8 \times 3.14} \times 17797.161 + 0.003867 \times 533\right]$$

$$1-\alpha = 0.95 \quad \Rightarrow \quad \alpha = 0.05$$

(c) Using GW190814 [15]

For Primary black hole, M =23.2 , $\chi = 0.07$

$$k = \frac{\sqrt{1-0.07^2}}{2 \times 23.2 \times \left(1+\sqrt{1-0.07^2}\right)} = 0.01072$$

$$A = 8 \times 3.14 \times 23.2^2 \times \left(1+\sqrt{1-0.07^2}\right) = 26905.97$$

$$\Omega = \frac{0.07}{2 \times 23.2 \times \left(1+\sqrt{1-0.07^2}\right)} = 0.000758$$

$$J = 0.07 \times 23.2^2 = 37.6768$$

For final black hole, M =25.6, χ = 0.28

$$A = 8 \times 3.14 \times 25.6^2 \times \left(1 + \sqrt{1 - 0.28^2}\right) = 32266.7807$$

$$J = 0.28 \times 25.6^2 = 183.5008$$

Now

$$\Delta M = 2.4\ ,\ \Delta A = 5360.81\ ,\ \Delta J = 145.8232$$

From the first law of black hole

$$2.4 = (1 - \alpha)\left[\frac{0.01072}{8 \times 3.14} \times 5360.81 + 0.000758 \times 145.8232\right]$$

$$1 - \alpha = 1 \Rightarrow \ \alpha = 0$$

These results are presented in Table 1.

Table 1: Deviation from the first law of black hole mechanics using three events [16-18]

Gravitational wave Event	Type of Event	Masses of primary and secondary stars ($M_{\odot}$)	Spin of primary black hole (χ)	Mass of final black hole ($M_{\odot}$)	Spin of final black hole (χ)	Mass Ratio of primary to secondary stars	Deviation from the first law of black hole mechanics ($\boldsymbol{\alpha}$)
GW151226	BBH	(14.2, 7.5)	0.4	20.8	0.74	1.89	~ 0.07
GW190412	BBH	(30.1, 8.3)	0.44	37.3	0.67	3.6	~ 0.05
GW190814	BH and a compact object	(23.2, 2.6)	0.07	25.6	0.28	8.92	~ 0

6. Conclusion

Two black holes orbiting each other due to gravitational attraction can set up a binary system. Coalescence of two black holes of a binary system creates disturbance in space-time curvature which produces gravitational waves. The first experimental observation of gravitational wave signal by the coalescence of two black holes was made by LIGO detectors at Livingston and Hanford on 14th September, 2015. Before the detection of gravitational wave, electromagnetic radiation (microwave, radio waves) was the only tool to get information about the universe. But after the discovery of gravitational waves, it opens a new way to study the universe. The discovery is not just proof of gravitational waves but a strong confirmation of the existence of black holes. Gravitational waves and the first law of black hole mechanics are related by the fundamental ideas of general relativity. Black holes can exchange energy and angular momentum with their surroundings through gravitational waves, which affect their masses, spins and event horizons in accordance with the laws outlined by the black hole mechanics. Treating the collision of binary systems as a perturbation process, we can roughly test the first law and it is found that the deviation from the first law is related to the mass ratio of the gravitational wave event. Higher the mass ratio of the gravitational wave source, the more consistent is the first law of black hole mechanics [8,12].

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