Fractal Dimension-Based Classification of Retinal Images Using Statistical Analysis

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Abstract: In this paper, fractal analysis has been used for detection of retinal disorder which leads to sudden vision loss. The two retinal disorders which leads to complete and sudden vision loss are diabetic retinopathy and retinal vein occlusion. The paper concentrates on vision loss due to retinal vein occlusion. The two methods used for calculating fractal dimension needed for fractal analysis are box counting and mass-radius methods. The statistical analysis is also done using Wilcoxon signed rank test to prove that there is significant difference between the fractal dimension of occluded and normal retina.

Keywords: Fractal analysis, vein occlusion, image processing, Wilcoxon signed rank test.

1. Introduction

The problem of sudden vision loss among older people is very frequently observed. This problem arises mainly because of two disorders. They are Diabetic Retinopathy and Occlusion. Diabetic retinopathy is seen in diabetic patients. Occlusion is basically obstruction in blood flow through a vein. When an artery crosses over a vein, occlusion occurs leading to hemorrhage, and this results in sudden vision loss. There are three types of occlusions. They are Central Retinal Vein Occlusion (CRVO), Branch Retinal Vein Occlusion (BRVO) and Hemi Retinal Vein Occlusion (HRVO) [1], 2013). Out of these occlusions, Branch retinal vein occlusion (BRVO) is most commonly seen. The report concentrates on detection of branch retinal vein occlusion which leads to sudden vision loss.

Retina is made up of repeating pattern of arteries and veins. This repeating pattern is called as fractals. Fractals are self-similar and repeating structure. Fractal dimension is the ratio which determines how much the pattern has changed with the scale on which it is measured. Fractal dimension is the key parameter used for detecting retinal disorder which leads to sudden vision loss. This analysis is known as fractal analysis. The range of fractal dimension for occluded as well as normal retina is obtained in this paper.

The value of fractal dimension were obtained using mass-radius and box counting method. Mass-radius and box counting methods were implemented on both occluded as well as normal retina. The considerable difference between the value of fractal dimension of an occluded as well as normal retina is observed. This paper has a study of detection of branch retinal vein occlusion using fractal analysis. The database contains 10 fundus photographs of normal retina and 10 fundus photographs of branch retinal vein occluded retina.

The statistical analysis of results obtained by mass-radius and box counting method is been done using a non parametric test. The non-parametric test used is Wilcoxon signed rank test. The results obtained using Wilcoxon signed rank test shows that there is significant difference between the fractal dimension of normal and occluded retina.

2. Literature Review

Retinal vein occlusion is the obstruction to the flow of blood through veins. The risk factors like hypertension, diabetes, high blood pressure, glaucoma is seen in older patients and hence retinal vein occlusion is mostly seen in older patients [1]. Non-ischemic and ischemic are two types of retinal vein occlusion. Ischemia is restriction in blood flow resulting in shortage of oxygen and glucose needed for cellular metabolism. For fractal analysis of retina, fractal dimension plays a key role.

2.1 Fractals

On magnifying the basic structure, one can observe repeating level of details and each looks like a whole. So in this case, the smallest and basic pattern is a fractal. Tree can be considered as a example since if focused on smallest portion it resembles the whole tree [3]. Considering image of a retina, it is also made up of resembling pattern of arteries and veins and hence using fractal analysis, study of disorder becomes easy.

There is no fundamental equation that governs fractal geometry. The two methods that are used to determine fractal dimension are mass-radius method and box counting method. For a normal patients, fractal dimension value is same for left and right eye. To diagnose retinal disorder, the nature of arteries and veins plays a key role [3] Fractal dimension value obtained for diabetic patient is much higher than the fractal dimension value obtained for a normal patient [3] Fractal dimension was obtained using box counting method as it was considered more accurate method than mass-radius method. The fractal dimension value does not change with age/ gender, it just changes with the pattern of arteries and veins. All the retina disorders like diabetic retinopathy, proliferative retinopathy, occlusions will result in different fractal dimension value. Variation in the pattern of retina characterizes the disease. Scaling also plays a important role in calculating fractal dimension [4]. Diabetes results in serious deformation of arteries and veins of retina because of which the fractal dimension value is higher than normal [5]. If the retina have same fractal dimension, then lacunarity is the quantity to be measured which describes the characteristics of fractals of retina [6]. Also image resolution plays a key role in calculating fractal dimension [6]. Fractal analysis is found out as the new strategy for computer assisted automated detection and also for quantification of diabetic retinopathy [7]. Fourier fractal dimension on gray scale images eliminating the binary segmentation of the vessel pattern which is more prone to error even if carried out by human expert or by automated algorithms [8]. Fourier transform of the enhanced image is the key step [8]. The branching pattern in retina have two important characteristics that are self similarity and decreasing density [9].

2.2 Occlusions

When an artery crosses over a vein, occlusion occurs which leads to hemorrhage further leading to sudden vision loss. The common features of arteriovenous crossings includes proximity of thin walled veins and thick walled arteries , narrowing a venous lumen. The comparison of number of arteriovenous crossings in retina of patients with and without branch retinal vein occlusion shows that arteriovenous crossing is more common in branch retinal vein occlusion patients [10]. Fig. 1 shows vein is obstructed by an artery and hence hemorrhage has taken place.



Fig 1: Branch Retinal Vein Occlusion [19]

Mean platelet volume is the indicator of size and activity of platelets. It is found to be higher in patients with retinal vein occlusion. It can be used as a predictive tool used for identifying the risk of disorder (A. Sahin, M. Sahin et.al, 2013). Optical Computed Tomography (OCT) and Fundus Autofluoroscence (FAF) are the two imaging techniques used in examining occlusion. FAF is used to characterize the retinal disease whereas OCT shows uniform autofluoroscence in eyes (T.

Sekiryu, T. Iida, E. Sakai et.al, 2012). Fig. 2 shows images obtained by OCT and autofluoroscence imaging.

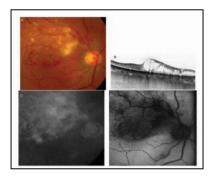


Fig 2: Fundus Photograph, OCT, Fluoroscein Angiograph and AutoFluoroscence images.

2.3 Fractal Analysis

Retina is made up of repeating pattern called fractals. Fractal Analysis has to be done to understand the repeating pattern of veins and arteries of retina of human eye. The Fractal dimension is the ratio at which the pattern changes with the scale at which it is measured. It is also said as a number that characterizes the distribution of branching vascular pattern in 2-Dimensional space [2]. The range of fractal dimension varies for occluded as well as normal retina. There are various methods to determine fractal dimension. They are mass-radius method and box counting method.

2.3.1 Mass Radius Method

Mass-radius method is been used in determining fractal dimension. The fractal dimension value can give lot of information related to retinal disorder. As the patient with diabetic retinopathy has fractal dimension value higher than the normal retina fractal dimension value [2]. In this method a series of circles of different radii centered about the optic nerve is constructed and the number of occupied pixels within a circle of a given radius is counted. The logarithm of the pixel mass within a circle of a given radius is then plotted against the logarithm of the radius of the circle, the slope of the log-log plot is D, the fractal dimension of the pattern as depicted in Fig. 3.

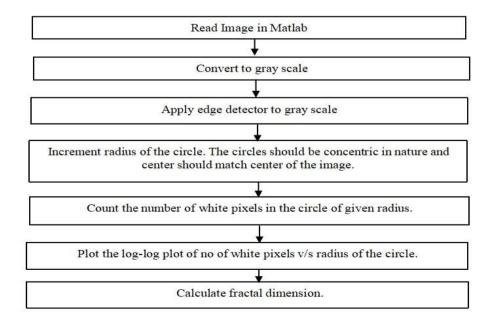


Fig. 3: Flowchart for Calculating fractal dimension using Mass radius method

The fractal dimension is calculated as

$$D = \frac{\ln \sum (pixel \ mass \ within \ the \ circle)}{\ln \sum (Radius \ of \ circle)}$$
(1)

Where D is fractal dimension. Using formula (1), fractal dimension can be calculated.

2.3.2 Box Counting Method

In this the fractal dimension was calculated by Box-counting method, since the pattern of retinal vessels are irregular [3]

Higher the fractal dimension value, more severity in diabetic retinopathy is seen.

In this method a series of boxes of decreasing size are constructed and the number of occupied pixels within a box of a given size is counted. The logarithm of the pixel mass within a box of a given size is then plotted against the logarithm of the size of box, the slope of the log-log plot is D, the fractal dimension of the pattern [3] In Fig. 4, box counting method is applied to a region.

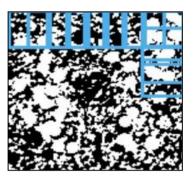


Fig. 4: Box counting method applied to region

Box counting method is been implemented on Fig. 4 and from the slope of the graph of natural log of size of box and number of boxes containing white pixels, fractal dimension is calculated. Fig 5 shows the flowchart for Box counting method and Fig 6 shows the obtained graph of the same.

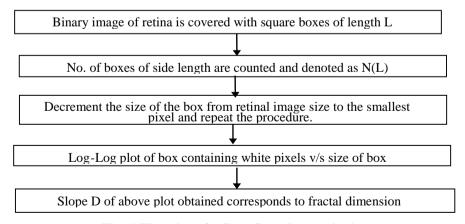


Fig. 5 Flowchart for Box Counting method

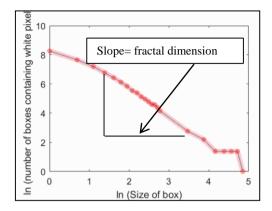


Fig 6: Plot of natural log of size of box and number of boxes containing white pixels.

In box counting method, fractal dimension is obtained from the slope of the graph. The fractal dimension obtained for Fig. 4 is 1.97.

3. Implementation

The retinal database was obtained from Prism Eye institute and Research Center, Navi Mumbai. All the results were obtained using MATLAB software. Fig.7 shows the occluded retina along with the gray scale image and blood vessel pattern of retina.

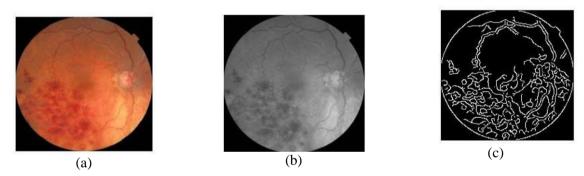


Fig 7: (a) Occluded retinal image, (b) Gray scale image and (c) blood vessel pattern of retina

Mass-radius method is applied to the occluded retina and Fig. 8 shows the output images obtained after applying mass-radius method. Similarly, mass-radius method was obtained on normal retina also.

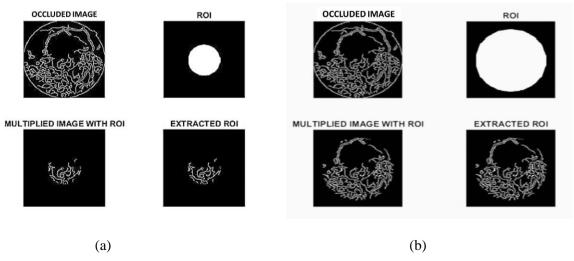


Fig 7: Output of mass-radius having region of interest 40 pixels and 80 pixels respectively.

Box counting method was also applied on Fig. 4, the output images obtained are shown in Fig 8.

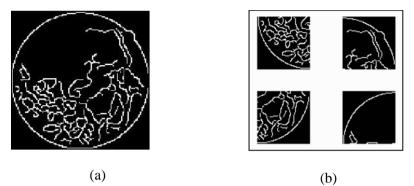


Fig 8: Blood vessel pattern of occluded retinal image and number of boxes containing white pixels having box size 64 pixels.

Table 2 and 3 shows the value of fractal dimension obtained using mass-radius and box counting method respectively.

Table 2: Output of fractal dimension using mass-radius method

Image No.	Fractal dimension of branch retinal vein occluded retina	Fractal dimension of normal retina	Image No.	Fractal dimension of branch retinal vein occluded retina	Fractal dimension of normal retina
1	1.6	1.5	6	1.58	1.48
2	1.68	1.47	7	1.58	1.44
3	1.64	1.46	8	1.55	1.43
4	1.61	1.5	9	1.55	1.45
5	1.65	1.42	10	1.56	1.46

Table 3: Output of fractal dimension using box counting method

Image No.	Fractal dimension of branch retinal vein occluded retina	Fractal dimension of normal retina	Image No.	Fractal dimension of branch retinal vein occluded retina	Fractal dimension of normal retina
1	1.57	1.43	6	1.54	1.46
2	1.53	1.44	7	1.55	1.39
3	1.54	1.39	8	1.54	1.47
4	1.55	1.48	9	1.53	1.39
5	1.52	1.47	10	1.53	1.45

3.1 Wilcoxon Signed Rank Test

Wilcoxon sign test is non-parametric test. It compares two dependent samples. Wilcoxon pools all the differences, ranks them and applies a negative sign to all the ranks where the differences between two observations is negative. This is known as signed rank. Wilcoxon has null hypothesis that both samples are from same population. It creates pool ranking of all observed differences between two dependent measurements.

Wilcoxon method is used on the values of fractal dimension obtained by both mass-radius method and box counting method. Wilcoxon test shows that fractal dimension for normal retina is less than fractal dimension of occluded retina. Table 4 shows the critical value table defined for alpha 0.05 and 0.01 for one tailed and two tailed respectively. The sample size (n) is from 5 to 30. Fig. 9 and Fig. 10 shows the results of Wilcoxon signed rule test applied on values of fractal dimension obtained by mass radius method and box counting method respectively. There was significant difference seen in fractal dimension of occluded and normal retina and null hypothesis is rejected as test statistics is more extreme than critical value in both mass-radius and box counting method. Table 4 shows the Wilcoxon Signed Rank Table.

n	Two-Ta	iled Test	One-Tailed Test		
n	$\alpha = .05$	$\alpha = .01$	$\alpha = .05$	$\alpha = .01$	
5			0		
6	0		2		
7	2		3	0	
8	3	0	5	1	
9	5	1	8	3	
10	8	3	10	5	

Table 4: Wilcoxon Signed Rank Table [11]

1.57 1.53 1.54 1.55 1.52 1.54	1.43 1.44 1.39 1.48 1.47	-1 -1 -1 -1	0.14 0.09 0.15 0.07	-7.5 -6 -9	Positive Sum	0
1.54 1.55 1.52	1.39 1.48	-1 -1	0.15	-	Positive Sum	0
1.55 1.52	1.48	-1		-9	Positive Sum	0
1.52			0.07			•
	1.47		-101	-2.5	Negative Sum	-55
1 54		-1	0.05	-1	Test Statistics	55
1.04	1.46	-1	0.08	-4.5	Critical Value	8 (2 tailed test, alpha= 0.05)
1.55	1.39	-1	0.16	-10		
1.54	1.47	-1	0.07	-2.5		
1.53	1.39	-1	0.14	-7.5		
1.53	1.45	-1	0.08	-4.5		
ere is no difference b	etween Fractal dimension					
ere is significant diffe	erence between Fractal d	imension	of occluded and I	Normal Retina		
atatistic > Critical Val	ue we reject FO					
ore There is significa	nt difference between Fi	ractal dim	ension of occlude	d and normal retina		
•	1.54 1.53 1.53 1.53 ere is no difference bere is significant diffe	1.54 1.47 1.53 1.39 1.53 1.45 ere is no difference between Fractal dimensicere is significant difference between Fractal d statistic > Critical Value, we reject FO	1.54 1.47 -1 1.53 1.39 -1 1.53 1.45 -1 ere is no difference between Fractal dimension of occlurere is significant difference between Fractal dimension statistic > Critical Value, we reject F0	1.54 1.47 -1 0.07 1.53 1.39 -1 0.14 1.53 1.45 -1 0.08 ere is no difference between Fractal dimension of occluded and Normal ere is significant difference between Fractal dimension of occluded and Normal statistic > Critical Value, we reject F0	1.54	1.54

Fig 9: Results of Wilcoxon signed rank test applied on values of fractal dimension obtained using massradius method

Sr No	FD of Occluded Retina	FD of Normal Retina	Sign	Absolute rank	Signed Rank		
1	1.6	1.5	-1	0.1	-2.5		
2	1.68	1.47	-1	0.21	-9		
3	1.64	1.46	-1	0.18	-8	Positive Sum	0
4	1.61	1.5	-1	0.11	-5	Negative	-55
5	1.65	1.42	-1	0.23	-10	Test Statistics	55
6	1.58	1.48	-1	0.1	-2.5	Critical Value	8 (2 tailed test, alpha= 0.05)
7	1.58	1.44	-1	0.14	-7		
8	1.55	1.43	-1	0.12	-6		
9	1.55	1.45	-1	0.1	-2.5		
10	1.56	1.46	-1	0.1	-2.5		
	F0: There is no difference b	petween Fractal dimension	etina				
	F1: There is significant diffe	erence between Fractal dir	ormal Retina				
	If Test statistic > Critical Va	lue, we reject F0					
	Therefore There is significa	ant difference between Fra	ctal dime	nsion of occluded	and normal retina		

Fig 10: Results of Wilcoxon signed rank test applied on values of fractal dimension obtained using box counting method

4. Conclusion

Retinal Vein Occlusion cases were diagnosed and these eyes were evaluated using fractal imaging of their corresponding retinal images. Further analysis was done by comparison of fractal analysis of retinal area involved and its blood vessel pattern versus the not involved retina and its blood vessel pattern. The use of fractal dimension has been successfully demonstrated for detection of branch retinal vein occlusion. The value of fractal dimension has been obtained for branch retinal vein occluded retina as well as normal retina. The value of fractal dimension has been obtained using mass-radius method and box counting method.

There was significant difference seen in the value of fractal dimension of occluded retina and normal retina using mass-radius and box counting method. The range of fractal dimension obtained for occluded retina using mass-radius method is 1.52 to 1.57. The range of fractal dimension obtained for normal retina using mass-radius method is 1.39 to 1.47. The range of fractal dimension obtained for occluded retina using box counting method is 1.55 to 1.68 and that for normal retina is 1.44 to 1.5. The range of fractal dimension obtained for maximum of upper and lower half of occluded retina is 1.52 to 1.74 while that for normal retina it is 1.33 to 1.48. Also using Wilcoxon signed rank test, It showed negative differences as fractal dimension of occluded retina will always be higher than fractal dimension of normal retina. Wilcoxon signed rank test hence proved that there is significant difference in fractal dimension obtained for occluded and normal retina.

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