Cover Domination Value of Connected Graphs

V. G. Bhagavathi Ammal

Department of Mathematics, S.T. Hindu College, Nagercoil, Kanyakumari District 629002

Abstract

Graph theory plays a vital part in real life applications like network design, network security, network monitoring, resource allocation, Facility location, location problems, network analysis like social network analysis, optimization problems etc. The domination concept designs effective and efficient networks where minimum number of nodes can dominate or control or regularize the entire network system. In network security system the dominating set can help the security resources strategically. In domination theory each vertex is capable of protecting the neighbouring vertices, it may be 1vertex or 2 vertices or 3 vertices or…… $(n-1)$ vertices. In this paper, we define a Cover Domination value (CDV) of graphs whereas the possibility of the number of vertices dominates two vertices simultaneously is known as 2 – cover domination value of G and is denoted by $dom^{\ 2\alpha}(G)$, the possibility of the number of vertices dominates three vertices simultaneously is known as 3 – cover domination value of G and is denoted by $dom^{\ 3\alpha}(G)$, and the possibility of the number of vertices dominates $n-1$ vertices simultaneously is known as $(n-1)$ – cover domination value of G and is denoted by $dom^{\ (n-1)\alpha}(G)$. And total of this cover domination value is known as Total Cover Domination value of a graph and is denoted by $T_{dom^{(n-1)\alpha}}(G)$ and the average of Total Cover domination Value of a graph is denoted by $CDV_{n-1}(G) = \frac{T_{dom^{(n-1)\alpha}}(G)}{nC_2}, for\ n = 3,4,\ldots\ldots,n$. For the trivial graph, cover domination value does not exist. Also, we found the cover domination value for general graphs and some standard graphs.

Key Words: Network analysis, adjacent vertices, connected graphs, cover domination value, total cover domination value, average cover domination value.

AMS Subject Classification: 05C69

1. INTRODUCTION

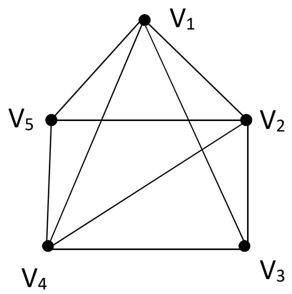
A simple connected graph G is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct lines of G, called edges. The vertex set and the edge set of G are denoted by $V(G) = n\ and\ E(G) = m$ respectively. A graph with n vertices and m edges is called a(n, m) graph. The degree of a vertex v in a graph G is the number of edges of G incident with v and is denoted by $deg(v)$. A graph G is r - regular if and only if every vertex of G has degree r. A graph G is complete if every pair of its vertices is adjacent. A complete graph on n vertices is denoted by K_n. A bipartite graph is a graph whose vertex $V\ (G)$ can be partitioned into two non-empty subsets $V_1\ and\ V_2$ such that every edge of G has one end in V_1 and other end in V_2; V_1 , V_2 is called a bipartition of G and is denoted by $K_{n,m}$.A graph is acyclic, if it has no cycles. A tree is a connected acyclic graph. In graph theory, some parameters have been studied widely such as connectivity, edge-connectivity, vertex covering, edge covering and domination etc. These parameters are considered to protect the neighboring vertices as well as edges. In a graph, every vertex is required to be protected and each vertex is capable of protecting every other vertex is called domination theory. A set $S \subseteq V(G)$ is said to be a **dominating set** in G, if every vertex in $V - S$ is adjacent to at least one vertex in S. The **domination number** of G is the minimum cardinality taken over all dominating sets in G and is denoted by $\gamma(G)$.

2. COVER DOMINATION VALUE OF CONNECTED GRAPHS

2.1 2 - cover domination value of graphs:

Definition 2.1.1: For any connected graph G, a *2 - Cover Domination value (CDV_2)* of graphs is defined as possibility of the number of vertices dominates two vertices simultaneously and *is denoted by* $dom^{2\alpha}(G)$. The total of 2 – cover domination value is said to be *Total 2 – cover domination value* of a graph and is denoted by $T_{dom^{2\alpha}}(G) = \sum_{\substack{i=1 \\ j=1}}^{n} dom^{2\alpha}(v_i, v_j)$. The average of Total 2 - cover domination value of graph is known as *Average 2 - cover domination value* of G and is denoted by $CDV_2(G) = \frac{T_{dom^{2\alpha}}(G)}{nC_2}$.

Example 2.1.2:



G

For G, $dom^{2\alpha}$ (v_1, v_2) = 4, $dom^{2\alpha}$ (v_1, v_3) = 3, $dom^{2\alpha}$ (v_1, v_4) = 4 , $dom^{2\alpha}$ (v_1, v_5) = 3, $dom^{2\alpha}$ (v_2, v_3) = 3, $dom^{2\alpha}$ (v_2, v_4) = 4 , $dom^{2\alpha}$ (v_2, v_5) = 3, $dom^{2\alpha}$ (v_3, v_4) = 3, $dom^{2\alpha}$ (v_3, v_5) = 3, $dom^{2\alpha}$ (v_4, v_5) = 3, thus $T_{dom^{2\alpha}}(G) = \sum_{\substack{i=1 \\ j=1}}^{n} dom^{2\alpha}(v_i, v_j) = 33$.

Hence $CDV_2(G) = \frac{T_{dom^{2\alpha}}(G)}{nC_2} = \frac{33}{5C_2} = \frac{33}{10} = 3.3$

Remark 2.1.3: For any connected graph $G = (V, E)$, the total number of vertices that dominates each and every pair of vertices is the sum of internally disjoint paths of length 1 or 2

Remark 2.1.4: The number of possible ways to find $dom^{(n-1)\alpha}, n = 3,4,5, \ldots\ldots, n$ of any graph $G(V, E)$ having n *vertices and* m *edges* is nC_r where $nC_r = \frac{n!}{r!(n-r)!}$, the same formula is applicable everywhere used here.

Definition 2.1.5: For any connected graph G and $v_1, v_2 \in V(G)$ if v_1 and v_2 are adjacent then they dominate each other, so $dom^{2\alpha}(v_1, v_2) = 1$.

Remark 2.1.6: $dom^{2\alpha}(G) = 0$ when $\deg(u) = 1, \forall\, u \in V(G)$

Theorem 2.1.7: For any connected graph G, the total number possibilities to dominate each pair of vertices is defined as

$$T_{dom^{2\alpha}}(G) = \begin{cases} 2n & if\ G \cong C_n \\ 2n-3 & if\ G \cong P_n \\ \sum_{u\in V(G)}[\deg(u)C_2] + m & if\ \deg(u) \geq 2 \\ 0 & if\ \deg(u) = 1 \end{cases}$$

And the average of 2 - cover domination value of the graph is obtained by

$$CDV_2(G) = \frac{T_{dom^{2\alpha}}(G)}{nC_2} = \begin{cases} \frac{4}{n-1} & if\ G \cong C_n \\ \frac{2n-3}{nC_2} & if\ G \cong P_n \\ \frac{\sum_{u\in V(G)}[\deg(u)C_2] + m}{nC_2} & if\ \deg(u) \geq 2 \\ 0 & if\ \deg(u) = 1 \end{cases}.$$

Proof: By remark 2.1.6, $dom^{2\alpha}(G) = 0$ when $\deg(u) = 1, \forall\, u \in V(G)$

Consider $\deg(V(G)) \geq 2$

$$dom^{2\alpha}(v_1, v_2) = \begin{cases} 1 & if\ v_1\ is\ adjacent\ with\ v_2 \\ 0 & if\ v_1\ is\ not\ adjacent\ with\ v_2 \end{cases} \quad (\because \text{by definition 2.1.5})$$

Consider $\deg(v_i) = 2,\ i = 1,2,\dots,n$ then $G \cong C_n$, then number of possibilities to get $T_{dom^{2\alpha}}(G)$ is $n\ times\ 2C_2$,

$$ie, dom^{2\alpha}(v_1, v_2) = dom^{2\alpha}(v_2, v_3) = dom^{2\alpha}(v_3, v_4) = \dots = dom^{2\alpha}(v_n, v_1) = 1 = 2C_2$$

$$\sum_{\substack{u\in V(G) \\ i=1\ to\ n}} dom^{2\alpha}(v_i, v_{i+1}) = 1 + 1 + \dots + 1 = n \quad (\because \text{by definition 2.1.5})$$

$$= \sum_{u\in V(G)}[2C_2] = n[2C_2]$$

$$= \sum_{u\in V(G)}[\deg(u)C_2]$$

Also, $dom^{2\alpha}(v_1, v_3) = dom^{2\alpha}(v_2, v_4) = dom^{2\alpha}(v_3, v_5) = \dots = dom^{2\alpha}(v_{n-1}, v_1) = 1 = 2C_2$.

$$\sum_{\substack{u\in V(G) \\ i=1\ to\ n}} dom^{2\alpha}(v_i, v_{i+2}) = 1 + 1 + \dots + 1 = n$$

$= \sum_{u\in V(G)}[2C_2] = n\,[2C_2] = \sum_{u\in V(G)}[\deg(u)C_2]$ ($\because$ (v_1, v_3) are dominated by v_2, (v_2, v_4) are dominated by v_3,….., (v_{n-1}, v_1) are dominated by v_n)

And $dom^{2\alpha}(v_1, v_4) = dom^{2\alpha}(v_1, v_5) = dom^{2\alpha}(v_1, v_6) = \dots = dom^{2\alpha}(v_{n-1}, v_n) = 0$

$dom^{2\alpha}(v_2, v_5) = dom^{2\alpha}(v_2, v_6) = dom^{2\alpha}(v_2, v_7) = \dots = dom^{2\alpha}(v_{n-2}, v_2) = 0$ . . . . . (since (v_1, v_4) is not dominated by a common vertex etc)

That is, $\sum_{\substack{u \in V(G) \\ i=1 \, to \, n}} dom^{\,2\alpha}(v_i, v_{i+3}) = \sum_{\substack{u \in V(G) \\ i=1 \, to \, n}} dom^{\,2\alpha}(v_i, v_{i+4}) = \ldots.. = \sum_{\substack{u \in V(G) \\ i=1 \, to \, n}} dom^{\,2\alpha}(v_i, v_{n-3}) = 0$

Hence $T_{dom^{2\alpha}}(G) = \sum_{\substack{u \in V(G) \\ i=1 \, to \, n}} dom^{\,2\alpha}(v_i, v_{i+1}) + \sum_{\substack{u \in V(G) \\ i=1 \, to \, n}} dom^{\,2\alpha}(v_i, v_{i+2}) = n + n = 2n$
$\therefore \sum_{u \in V(G)}[\deg(u) C_2] + m \quad if \deg(u) = 2$. Thus, the average of 2 – cover domination value of graph is $CDV_2(G) = \frac{T_{dom^{2\alpha}}(G)}{nC_2} = \frac{2n}{nC_2} = \frac{2n}{\left(\frac{n(n-1)}{2}\right)} = \frac{4}{n-1}$. For instance consider $G \cong C_4$, the number of vertices dominates both v_1, v_2 is 1, therefore, $dom^{\,2\alpha}$ (v_1, v_2) = 1, the number of vertices dominates both v_1, v_3 is 2, therefore, $dom^{\,2\alpha}$ (v_1, v_3) = 2, similarly it's easy to find the 2 – cover domination value for the remaining vertices. Hence, $dom^{\,2\alpha}$ (v_1, v_4) = 1, $dom^{\,2\alpha}$ (v_2, v_3) = 1, $dom^{\,2\alpha}$ (v_2, v_4) = 2, $dom^{\,2\alpha}$ (v_3, v_4) = 1, Hence $CDV_2(G) = \frac{T_{dom^{2\alpha}}(G)}{nC_2} = \frac{4}{3}$
By above theorem, $T_{dom^{2\alpha}}(G) = \sum_{u \in V(G)}[\deg(u) C_2] + m = 4[2C_2] + 4 = 8$ ($\because in \, C_4$, 4 vertices having degree 2 and m = 4)
Thus,$CDV_2(G) = \frac{T_{dom^{2\alpha}}(G)}{nC_2} = \frac{8}{4C_2} = \frac{4}{3}$. Hence the theorem verified for the graph having degree 2.
Consider $G \cong P_n$, path with n vertices then number of possibilities to get

$T_{dom^{2\alpha}}(G)$ is $n - 2 \; times \; 2C_2$, $\because$ ($\deg(v_1) = \deg(v_n) = 1$ and by remark 2.16 $dom^{2\alpha} = 0$)

By Definition. 2.1.5,

$dom^{\,2\alpha}(v_1, v_2) = dom^{\,2\alpha}(v_2, v_3) = dom^{\,2\alpha}(v_3, v_4) = \ldots. = dom^{\,2\alpha}(v_{n-1}, v_n) = 1 \quad = 2C_2.$

$$\sum_{\substack{u \in V(G) \\ i=1 \, to \, n}} dom^{\,2\alpha}(v_i, v_{i+1}) = 1 + 1 + \cdots + 1 = n$$

$$= \sum_{u \in V(G)}[2C_2] = n[2C_2] = \sum_{u \in V(G)}[\deg(u) C_2]$$

Also, $dom^{\,2\alpha}(v_1, v_3) = dom^{\,2\alpha}(v_2, v_4) = dom^{\,2\alpha}(v_3, v_5) = \ldots. = dom^{\,2\alpha}(v_{n-2}, v_1) = 1 \quad = 2C_2$

$$\sum_{\substack{u \in V(G) \\ i=1 \, to \, n}} dom^{\,2\alpha}(v_i, v_{i+2}) = 1 + 1 + \cdots + 1 = n$$

$$= \sum_{u \in V(G)}[2C_2] = n\,[2C_2] = \sum_{u \in V(G)}[\deg(u) C_2]$$

Since it is a acyclic graph then $dom^{\,2\alpha}(v_1, v_2) = dom^{\,2\alpha}(v_1, v_3) = 1 \; and \; dom^{\,2\alpha}(v_1, v_4) = dom^{\,2\alpha}(v_1, v_5) = \ldots.. = dom^{\,2\alpha}(v_1, v_n) = 0$

Similarly, $dom^{\,2\alpha}(v_2, v_3) = dom^{\,2\alpha}(v_2, v_4) = 1 \; and \; dom^{\,2\alpha}(v_2, v_5) = dom^{\,2\alpha}(v_2, v_6) = \ldots.. = dom^{\,2\alpha}(v_2, v_n) = 0$ etc.

Hence $T_{dom^{2\alpha}}(G) = \sum_{\substack{u \in V(G) \\ i=1 \, to \, n}} dom^{\,2\alpha}(v_i, v_{i+1}) + \sum_{\substack{u \in V(G) \\ i=1 \, to \, n}} dom^{\,2\alpha}(v_i, v_{i+2}) = n + n - 3 = 2n - 3$. Thus, the average of 2 – cover domination value of graph is $CDV_2(G) = \frac{T_{dom^{2\alpha}}(G)}{nC_2} =$

$\frac{2n-3}{nC_2}$. For instance consider $G \cong P_4$, the number of vertices dominates both v_1, v_2 is 1, therefore, $dom^{2\alpha}$ (v_1, v_2) = 1, the number of vertices dominates both v_1, v_3 is 1, therefore, $dom^{2\alpha}$ (v_1, v_3) = 1, similarly it's easy to find the 2 – cover domination value for the remaining vertices. Hence, $dom^{2\alpha}$ (v_1, v_4) = 0, $dom^{2\alpha}$ (v_2, v_3) = 1, $dom^{2\alpha}$ (v_2, v_4) = 1, $dom^{2\alpha}$ (v_3, v_4) = 1, Hence $CDV_2(G) = \frac{T_{dom^{2\alpha}}(G)}{nC_2} = \frac{5}{3}$

By above theorem, $T_{dom^{2\alpha}}(G) = \sum_{u \in V(G)}[\deg(u)C_2] + m = 2[2C_2] + 3 = 5$ ($\because$ $in\ P_4$, 2 vertices having degree 2 and m = 3)

Thus, $CDV_2(P_4) = \frac{T_{dom^{2\alpha}}(G)}{nC_2} = \frac{5}{4C_2} = \frac{5}{3}$.

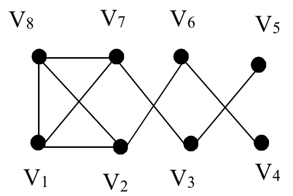
Hence the theorem verified for the graph having degree 2.

Similarly, if $\deg(v_1) = 3$ then then number of possibilities to get $T_{dom^{2\alpha}}(G)$ is $3C_2 + number\ of\ edges$, if $\deg(v_1) = 4$ then then number of possibilities to get $T_{dom^{2\alpha}}(G)$ is $4C_2\ + number\ of\ edges$ and hence,

$T_{dom^{2\alpha}}(G) = \sum_{u \in V(G)}[\deg(u)C_2]\ + m$. (since by definition 2.1.5).

Hence, $T_{dom^{2\alpha}}(G) = \sum_{u \in V(G)}[\deg(u)C_2] + m$. Thus $CDV_2(G) = \frac{T_{dom^{2\alpha}}(G)}{nC_2}$

Example 2.1.8:



G_1

For graph G_1, $dom^{2\alpha}$ (v_1, v_2) = 2, $dom^{2\alpha}$ (v_1, v_3) = 1, $dom^{2\alpha}$ (v_1, v_6) = 1, $dom^{2\alpha}$ (v_1, v_7) = 2, $dom^{2\alpha}$ (v_1, v_8) = 3, $dom^{2\alpha}$ (v_2, v_4) = 1, $dom^{2\alpha}$ (v_2, v_6) = 1, $dom^{2\alpha}$ (v_2, v_7) = 2, $dom^{2\alpha}$ (v_2, v_8) = 2 $dom^{2\alpha}$ (v_3, v_5) = 1, $dom^{2\alpha}$ (v_3, v_7) = 1, $dom^{2\alpha}$ (v_3, v_8) = 1, $dom^{2\alpha}$ (v_4, v_6) = 1, $dom^{2\alpha}$ (v_5, v_7) = 1, $dom^{2\alpha}$ (v_6, v_8) = 1, $dom^{2\alpha}$ (v_7, v_8) = 2. Hence $CDV_2(G) = \frac{23}{8C_2} = 0.82$

By above theorem,

$T_{dom^{2\alpha}}(G) = \sum_{v_i, v_j \in V(G)} dom^{2\alpha}(v_i, v_j) + m$ =4 $(3C_2) + 2\ (2C_2) + 9 = 23$.

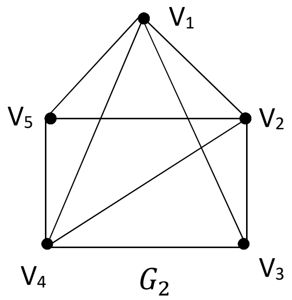
($\because$ 4 vertices of degree 3, 2 vertices of degree 2 and $dom^{2\alpha}$ value does not exist for degree < 2).

Thus, $CDV_2(G) = \frac{T_{dom^{2\alpha}}(G)}{nC_2} = \frac{23}{8C_2} = 0.82$

2.2 3 - cover domination value of graphs:

Definition 2.2.1: For any connected graph G, a *3 - Cover Domination value (CDV3)* of graphs is defined as possibility of the number of vertices dominates three vertices simultaneously and is denoted by $dom^{3\alpha}(G)$. The total of 3 – cover domination value is said to be *total 3 – cover domination value* of a graph and is denoted by $T_{dom^{3\alpha}}(G) = \sum^{n}_{\substack{i=1 \\ j=1, k=1}} dom^{3\alpha}(v_i, v_j, v_k)$. The average of 3 - cover domination value of graph is known as *Average 3 - cover domination value of G* and is denoted by $CDV_3(G) = \frac{T_{dom^{3\alpha}}(G)}{nC_2}$.

Example 2.2.2:



For G2, $dom^{3\alpha}$ (v_1, v_2, v_3) = 1, $dom^{3\alpha}$ (v_1, v_2, v_4) = 2, $dom^{3\alpha}$ (v_1, v_2, v_5) =1, $dom^{3\alpha}$ (v_1, v_3, v_4) =1, $dom^{3\alpha}$ (v_1, v_3, v_5) =2, $dom^{3\alpha}$ (v_1, v_4, v_5) =1, $dom^{3\alpha}$ (v_2, v_3, v_4) =1, $dom^{3\alpha}$ (v_2, v_3, v_5) =2, $dom^{3\alpha}$ (v_2, v_4, v_5) =1, $dom^{3\alpha}$ (v_3, v_4, v_5) =2. Hence $CDV_3(G_1) = \frac{14}{5C_2} = \frac{14}{10} = 1.4$

Remark 2.2.3: $dom^{3\alpha}(G) = 0$ when $\deg(u) \leq 2, \forall\, u \in V(G)$

Theorem 2.2.4: For any connected graph G, the total 3 - cover domination value of G is,

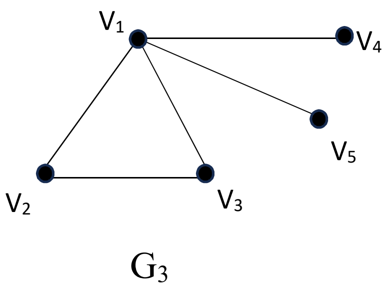
$$T_{dom^{3\alpha}}(G) = \begin{cases} \sum_{u \in V(G)}[\deg(u)C_3] & if \deg(u) \geq 3 \\ 0 & if \deg(u) \leq 2 \end{cases}$$

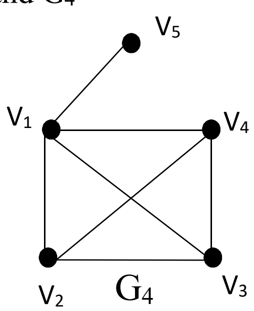
And the average of 3 - cover domination value of the graph is obtained by

$$CDV_3(G) = \frac{T_{dom^{3\alpha}}(G)}{nC_2} = \begin{cases} \frac{\sum_{u \in V(G)}[\deg(u)C_3]}{nC_2} & if\ if \deg(u) \geq 3 \\ 0 & if \deg(u) \leq 2 \end{cases}.$$

Proof: For Graph G, $deg(G) \leq 2$, it is obvious that $dom^{3\alpha} = 0$. Consider $deg(G) \geq 3$. If any vertex is of degree 3 then there is only one possibility to dominate 3 vertices. ie, if $deg(V(G)) = 3$ then $dom^{3\alpha} = 1 = 3C_3$. Consider a graph with $v_1 \in G$ having degree 4 is adjacent to v_2, v_3, v_4, v_5 then the number of possibilities to dominate 3 vertices is $(v_2, v_3, v_4), (v_2, v_3, v_5), (v_2, v_4, v_5)$ *and* (v_3, v_4, v_5). Thus, $dom^{3\alpha} = 4 = 4C_3$. The total 3 – cover domination value is obtained by summing up the number of possibilities of every vertex of G. Thus, $T_{dom^{3\alpha}}(G) = \begin{cases} \sum_{u \in V(G)}[\deg(u)C_3] & if \deg(u) \geq 3 \\ 0 & if \deg(u) \leq 2 \end{cases}$. The average of 3 – cover domination value is obtained by $CDV_3(G) = \frac{T_{dom^{3\alpha}}(G)}{nC_2}$.

Example 2.2.5: Consider the following graphs G3 and G4





In G_3 , $dom^{3\alpha}(v_1, v_2, v_3) = 0$, $dom^{3\alpha}(v_1, v_2, v_4) = 0$, $dom^{3\alpha}(v_1, v_2, v_5) = 0$
$dom^{3\alpha}(v_1, v_3, v_4) = 0$, $dom^{3\alpha}(v_1, v_3, v_5) = 0$, $dom^{3\alpha}(v_1, v_4, v_5) = 0$,$dom^{3\alpha}(v_2, v_3, v_4) = 1$
$dom^{3\alpha}(v_2, v_3, v_5) = 1$, $dom^{3\alpha}(v_2, v_4, v_5) = 1$ $dom^{3\alpha}(v_3, v_4, v_5) = 1$.

Therefore, the 3-cover domination value of G_3 is, $CDV_3(G_3) = \frac{T_{dom^{3\alpha}}(G)}{nC_2} = \frac{4}{5C_2} = \frac{4}{10} = 0.4$

Using theorem 2.2.4, In G_3, $T_{dom^{3\alpha}}(G) = 4C_3 = 4$ and $CDV_3(G_3) = \frac{T_{dom^{3\alpha}}(G)}{nC_2} = \frac{4}{5C_2} = \frac{4}{10} = 0.4$

(since $dom^{3\alpha} = 0 \; if \deg(u) \leq 2$)

In G_4 , $dom^{3\alpha}(v_1, v_2, v_3) = 1$, $dom^{3\alpha}(v_1, v_2, v_4) = 1$, $dom^{3\alpha}(v_1, v_2, v_5) = 1$
$dom^{3\alpha}(v_1, v_3, v_4) = 1$, $dom^{3\alpha}(v_1, v_3, v_5) = 1$, $dom^{3\alpha}(v_1, v_4, v_5) = 0$, $dom^{3\alpha}(v_2, v_3, v_4) = 1$
$dom^{3\alpha}(v_2, v_3, v_5) = 1$, $dom^{3\alpha}(v_2, v_4, v_5) = 0$ $dom^{3\alpha}(v_3, v_4, v_5) = 0$

Therefore, the 3-cover domination value of G_4 is,

$$CDV_3(G_4) = \frac{T_{dom^{3\alpha}}(G)}{nC_2} = \frac{7}{5C_2} = \frac{7}{10} = 0.7$$

Using theorem 2.2.4, In G_4, $T_{dom^{3\alpha}}(G) = 4C_3 + 3.(3C_3) = 7$ and

$$CDV_3(G_4) = \frac{T_{dom^{3\alpha}}(G)}{nC_2} = \frac{7}{5C_2} = \frac{7}{10} = 0.7$$

Comparing G_3 and G_4, $CDV_3(G_3) < CDV_3(G_4)$. Thus, G_4 is the preferred graph.

2.3 $(n-1)$ - cover domination value of graphs

Definition 2.3.1: For any connected graph G, a $(n-2)$ - Cover Domination value (CDV_{n-2}) of graphs is defined as possibility of the number of vertices dominates $(n-2)$ vertices simultaneously and is denoted by $dom^{(n-2)\alpha}(G)$. The total of $(n-2)$ – cover domination value is said to be *Total $(n-2)$ – cover domination value* of a graph and is denoted by $T_{dom^{(n-2)\alpha}}(G) = \sum dom^{(n-2)\alpha}(v_1, v_2, \dots, v_{n-2})$. The average of $(n-2)$ - cover domination value of graph is known as *Average $(n-2)$ - cover domination value* of G and is denoted by $CDV_{(n-2)}(G) = \frac{T_{dom^{(n-2)\alpha}}(G)}{nC_2}$

Definition 2.3.2: For any connected graph G, a $(n-1)$ - Cover Domination value (CDV_{n-1}) of graphs is defined as possibility of the number of vertices dominates $n-1$ vertices simultaneously and is denoted by $dom^{(n-1)\alpha}(G)$. The total of $(n-1)$ – cover domination value is said to be *total $(n-1)$ – cover domination value* of a graph and is denoted by $T_{dom^{(n-1)\alpha}}(G) = \sum dom^{(n-1)\alpha}(v_1, v_2, \dots, v_{n-1})$. The average of $(n-1)$ - cover domination value of graph is known as *Average $(n-1)$ - cover domination value* of G and is denoted by $CDV_{(n-1)}(G) = \frac{T_{dom^{(n-1)\alpha}}(G)}{nC_2}$.

Remark 2.3.3: $dom^{(n-1)\alpha}(G) = 0$ when $\deg(u) \leq n-2, \forall\, u \in V(G)$

Theorem 2.3.4: For any connected graph G,

$$T_{dom^{(n-1)\alpha}}(G) = \begin{cases} \sum_{u \in V(G)}[\deg(u)C_{n-1}] & if \deg(u) = n-1 \\ 0 & if \deg(u) \leq n-2 \end{cases}$$

And the average of $(n-1)$ - cover domination value of the graph is obtained by $CDV_{(n-1)}(G) = \frac{T_{dom^{(n-1)\alpha}}(G)}{nC_2} = \begin{cases} \frac{\sum_{u \in V(G)}[\deg(u)C_{n-1}]}{nC_2} & if\ if \deg(u) = n-1 \\ 0 & if \deg(u) \leq n-2 \end{cases}$.

Proof: For G, vertex's degree is less than or equal to $(n-2)$, it is obvious that $dom^{(n-1)\alpha}$ value is 0. Consider $V(G) = n-1$. Then the number of possibilities is one. ie, $(n-1)C_{(n-1)}$. For instance, consider the graph $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$, and v_1 *and* v_2 have degree 5. The number of possibilities to dominate 5 vertices simultaneously is $6C_5$ are $(v_1, v_2, v_3, v_4, v_5), (v_1, v_2, v_3, v_4, v_6), (v_1, v_2, v_3, v_5, v_6), (v_1, v_2, v_4, v_5, v_6), (v_1, v_3, v_4, v_5, v_6)$, $(v_2, v_3, v_4, v_5, v_6)$. Thus $dom^{(n-1)\alpha}(v_1, v_2, v_3, v_4, v_5) = 0, dom^{(n-1)\alpha}(v_1, v_2, v_3, v_4, v_6) = 0$, $dom^{(n-1)\alpha}(v_1, v_2, v_3, v_5, v_6) = 0$, $dom^{(n-1)\alpha}(v_1, v_2, v_4, v_5, v_6) = 0$, $dom^{(n-1)\alpha}(v_1, v_3, v_4, v_5, v_6) = 1$ and $dom^{(n-1)\alpha}(v_2, v_3, v_4, v_5, v_6) = 1$.

Hence, $T_{dom^{(n-1)\alpha}}(G) = \sum_{u \in V(G)}[\deg(u)C_{n-1}] \quad if \deg(u) = n-1$

$\therefore \quad T_{dom^{(n-1)\alpha}}(G) = 2$

Therefore, $CDV_{(n-1)}(G) = \frac{T_{dom^{(n-1)\alpha}}(G)}{nC_2} = \frac{2}{6C_2} = \frac{2}{15}$

Using theorem 2.3.4, $T_{dom^{(n-1)\alpha}}(G) = \sum_{u \in V(G)}[\deg(u)C_{n-1}] = 5C_5 + 5C_5 = 2$, since $\deg(u) < n-1$ is 0 and only two vertices having degree 5.

Therefore, $CDV_{(n-1)}(G) = \frac{T_{dom^{(n-1)\alpha}}(G)}{nC_2} = \frac{2}{6C_2} = \frac{2}{15}$

Corollary 2.3.4.1: For complete graph K_n,

(i) $T_{dom^{2\alpha}}(K_n) = \begin{cases} \frac{n(n-1)^2}{2} & if \deg(u) \geq 2 \\ 0 & if \deg(u) = 1 \end{cases}$ and $CDV_2(K_n) = n-1$

(ii) $T_{dom^{3\alpha}}(K_n) = \begin{cases} \frac{n(n-1)(n-2)(n-3)}{6} & if \deg(u) \geq 3 \\ 0 & if \deg(u) \leq 2 \end{cases}$ and $CDV_3(K_n) = \frac{(n-2)(n-3)}{3}$

(iii) $T_{dom^{4\alpha}}(K_n) = \begin{cases} \frac{n(n-1)(n-2)(n-3)(n-4)}{24} & if \deg(u) \geq 4 \\ 0 & if \deg(u) \leq 3 \end{cases}$ and

$CDV_4(K_n) = \frac{(n-2)(n-3)(n-4)}{12}$……….

(iv) $T_{dom^{(n-1)\alpha}}(K_n) = \begin{cases} n & if \deg(u) = n-1 \\ 0 & if \deg(u) < n-1 \end{cases}$ and $CDV_{n-1}(K_n) = \frac{2}{n-1}$

Proof of (i) By theorem 2.1.7,

$$T_{dom^{2\alpha}}(G) = \begin{cases} \sum_{u \in V(G)}[\deg(u)C_2] + m & if \deg(u) \geq 2 \\ 0 & if \deg(u) = 1 \end{cases}$$

For K_n, $\sum_{u \in V(G)}[\deg(u)C_2] + m = n\,[(n-1)C_2] + \frac{n(n-1)}{2}$

$$= \frac{n(n-1)(n-2)}{1.2} + \frac{n(n-1)}{2}$$
$$= \frac{n(n-1)}{2}(n-1)$$
$$= \frac{n\,(n-1)^2}{2}$$

Therefore, $CDV_2(K_n) = \frac{T_{dom^{2\alpha}}(G)}{nC_2} = \frac{\left(\frac{n\,(n-1)^2}{2}\right)}{\left(\frac{n(n-1)}{2}\right)} = n-1$

Proof of (ii) By theorem 2.2.4, For any connected graph G,

$$T_{dom^{3\alpha}}(G) = \begin{cases} \sum_{u \in V(G)}[\deg(u)C_3] & if \deg(u) \geq 3 \\ 0 & if \deg(u) \leq 2 \end{cases}$$

For K_n, $T_{dom^{3\alpha}}(G) = \sum_{u \in V(G)}[\deg(u)C_3] = n\,[(n-1)C_3]$

$$= \frac{n(n-1)(n-2)(n-3)}{1.2.3}$$

Therefore, $CDV_3(K_n) = \frac{T_{dom^{3\alpha}}(G)}{nC_2} = \frac{\left(\frac{n(n-1)(n-2)(n-3)}{6}\right)}{\left(\frac{n(n-1)}{2}\right)} = \frac{(n-2)(n-3)}{3}$

Proof of (iii) we have $T_{dom^{4\alpha}}(G) = \begin{cases} \sum_{u \in V(G)}[\deg(u)C_4] & if \deg(u) \geq 4 \\ 0 & if \deg(u) \leq 3 \end{cases}$

For K_n, $T_{dom^{4\alpha}}(G) = \sum_{u \in V(G)}[\deg(u)C_4] = \sum_{u \in V(G)}[(\mathrm{n}-1)\,C_4] = \frac{n(n-1)(n-2)(n-3)(n-4)}{1.2.3.4}$

Therefore, $CDV_4(K_n) = \frac{T_{dom^{4\alpha}}(G)}{nC_2} = \frac{\left(\frac{n(n-1)(n-2)(n-3)(n-4)}{24}\right)}{\left(\frac{n(n-1)}{2}\right)} = \frac{(n-2)(n-3)(n-4)}{12}$

Proof of (iv) we have $T_{dom^{(n-1)\alpha}}(G) = \begin{cases} \sum_{u \in V(G)}[\deg(u)C_{n-1}] & if \deg(u) = n-1 \\ 0 & if \deg(u) \leq n-2 \end{cases}$

For K_n, $T_{dom^{(n-1)\alpha}}(G) = \sum_{u \in V(G)}[(\mathrm{n}-1)\,C_{n-1}] = n$ (since n vertices having degree $n-1$)

Therefore, $CDV_{n-1}(K_n) = \frac{T_{dom^{(n-1)\alpha}}(G)}{nC_2} = \frac{n}{\left(\frac{n(n-1)}{2}\right)} = \frac{2}{n-1}$

Corollary 2.3.4.2: For star graph $K_{1,n-1}$,

(i) $T_{dom^{2\alpha}}(K_{1,n-1}) = \begin{cases} \frac{n(n-1)}{2} & if \deg(u) \geq 2 \\ 0 & if \deg(u) = 1 \end{cases}$ and $CDV_2(K_{1,n-1}) = 1$

(ii) $T_{dom^{3\alpha}}(K_{1,n-1}) = \begin{cases} (n-1)C_3 & if \deg(u) \geq 3 \\ 0 & if \deg(u) \leq 2 \end{cases}$ and $CDV_3(K_{1,n-1}) = \frac{(n-1)C_3}{nC_2}$

(iii) $T_{dom^{4\alpha}}(K_{1,n-1}) = \begin{cases} (n-1)C_4 & if \deg(u) \geq 4 \\ 0 & if \deg(u) \leq 3 \end{cases}$ and $CDV_4(K_{1,n-1}) = \frac{(n-1)C_4}{nC_2}$

. . . .

(iv) $T_{dom^{(n-1)\alpha}}(K_{1,n-1}) = \begin{cases} 1 & if \deg(u) = n-1 \\ 0 & if \deg(u) < n-1 \end{cases}$ and $CDV_{n-1}(K_{1,n-1}) = \frac{2}{n(n-1)}$

Proof of (i) By theorem 2.1.7,

$$T_{dom^{2\alpha}}(G) = \begin{cases} \sum_{u \in V(G)}[\deg(u)C_2] + m & if \deg(u) \geq 2 \\ 0 & if \deg(u) = 1 \end{cases}$$

For $K_{1,n-1}$, $\sum_{u \in V(G)}[\deg(u)C_2] + m = (n-1)C_2 + (n-1)$

$$= \frac{(n-1)(n-2)}{1.2} + (n-1)$$

$$= \frac{(n-1)(n-2)+2(n-1)}{2}$$

$$= \frac{n(n-1)}{2}$$

Therefore, $CDV_2(K_{1,n-1}) = \frac{T_{dom^{2\alpha}}(G)}{nC_2} = \frac{\left(\frac{n(n-1)}{2}\right)}{\left(\frac{n(n-1)}{2}\right)} = 1$

Proof of (ii) By theorem 2.2.4, For any connected graph G,

$$T_{dom^{3\alpha}}(G) = \begin{cases} \sum_{u \in V(G)}[\deg(u)C_3] & if \deg(u) \geq 3 \\ 0 & if \deg(u) \leq 2 \end{cases}$$

For $K_{1,n-1}$, $T_{dom^{3\alpha}}(G) = \sum_{u \in V(G)}[\deg(u)C_3] = (n-1)C_3$

Therefore, $CDV_3(K_{1,n-1}) = \frac{T_{dom^{3\alpha}}(G)}{nC_2} = \frac{(n-1)C_3}{nC_2}$

Proof of (iii) we have $T_{dom^{4\alpha}}(G) = \begin{cases} \sum_{u \in V(G)}[\deg(u)C_4] & if \deg(u) \geq 4 \\ 0 & if \deg(u) \leq 3 \end{cases}$

For $K_{1,n-1}$, $T_{dom^{4\alpha}}(G) = \sum_{u \in V(G)}[\deg(u)C_4] = \sum_{u \in V(G)}[(\mathrm{n}-1)\, C_4]$

Therefore, $CDV_4(K_{1,n-1}) = \frac{T_{dom^{4\alpha}}(G)}{nC_2} = \frac{(\mathrm{n}-1)\, C_4}{nC_2}$

Proof of (iv) we have $T_{dom^{(n-1)\alpha}}(G) = \begin{cases} \sum_{u \in V(G)}[\deg(u)C_{n-1}] & if \deg(u) \geq 4 \\ 0 & if \deg(u) \leq 3 \end{cases}$

For $K_{1,n-1}$, $T_{dom^{(n-1)\alpha}}(G) = \sum_{u \in V(G)}[(\mathrm{n}-1)\, C_{n-1}] = 1$ ($\because$ one vertex is of degree $n-1$)

Therefore, $CDV_{n-1}(K_{1,n-1}) = \frac{T_{dom^{(n-1)\alpha}}(G)}{nC_2} = \frac{1}{nC_2} = \frac{2}{n(n-1)}$

Corollary 2.3.4.3: For Wheel graph $W_{1,n-1}$,

(i) $T_{dom^{2\alpha}}(W_{1,n-1}) = \begin{cases} \frac{(n-1)(n+8)}{2} & if \deg(u) \geq 2 \\ 0 & if \deg(u) = 1 \end{cases}$ and $CDV_2(W_{1,n-1}) = \frac{n+8}{n}$

(ii) $T_{dom^{3\alpha}}(W_{1,n-1}) = \begin{cases} (n-1)\left(\frac{(n-2)(n-3)+6}{6}\right) & if \deg(u) \geq 3 \\ 0 & if \deg(u) \leq 2 \end{cases}$ and

$CDV_3(W_{1,n-1}) = \frac{(n-2)(n-3)+6}{3n}$

(iii) $T_{dom^{4\alpha}}(W_{1,n-1}) = \begin{cases} (n-1)C_4 & if \deg(u) \geq 4 \\ 0 & if \deg(u) \leq 3 \end{cases}$ and $CDV_4(W_{1,n-1}) = \frac{(n-1)C_4}{nC_2}$

. . . .

(iv) $T_{dom^{(n-1)\alpha}}(W_{1,n-1}) = \begin{cases} 1 & if \deg(u) = n-1 \\ 0 & if \deg(u) < n-1 \end{cases}$ and $CDV_4(W_{1,n-1}) = \frac{2}{n(n-1)}$

Proof of (i) By theorem 2.1.7,

$$T_{dom^{2\alpha}}(G) = \begin{cases} \sum_{u \in V(G)}[\deg(u)C_2] + m & if \deg(u) \geq 2 \\ 0 & if \deg(u) = 1 \end{cases}$$

For $W_{1,n-1}$, $\sum_{u \in V(G)}[\deg(u)C_2] + m = (n-1)C_2 + (n-1)(3C_2) + 2(n-1)$

$$= \frac{(n-1)(n-2)}{1.2} + 3(n-1) + 2(n-1)$$
$$= (n-1)\left(\frac{(n-2)}{2} + 5\right)$$
$$= \frac{(n-1)(n+8)}{2}$$

Therefore, $CDV_2(W_{1,n-1}) = \frac{T_{dom^{2\alpha}}(G)}{nC_2} = \frac{\left(\frac{(n-1)(n+8)}{2}\right)}{\left(\frac{n(n-1)}{2}\right)} = \frac{n+8}{n}$

Proof of (ii) By theorem 2.2.4, For any connected graph G,

$$T_{dom^{3\alpha}}(G) = \begin{cases} \sum_{u \in V(G)}[\deg(u)C_3] & if \deg(u) \geq 3 \\ 0 & if \deg(u) \leq 2 \end{cases}$$

For $W_{1,n-1}$, $T_{dom^{3\alpha}}(G) = \sum_{u \in V(G)}[\deg(u)C_3] = (n-1)C_3 + (n-1)(3C_3)$

$$= \frac{(n-1)(n-2)(n-3)}{1.2.3} + (n-1) = (n-1)\left(\frac{(n-2)(n-3)}{6} + 1\right) = (n-1)\left(\frac{(n-2)(n-3)+6}{6}\right)$$

Therefore, $CDV_3(W_{1,n-1}) = \frac{T_{dom^{3\alpha}}(G)}{nC_2} = \frac{(n-1)\left(\frac{(n-2)(n-3)+6}{6}\right)}{nC_2} = \left(\frac{(n-2)(n-3)+6}{3n}\right)$

Proof of (iii) we have $T_{dom^{4\alpha}}(G) = \begin{cases} \sum_{u \in V(G)}[\deg(u)C_4] & if \deg(u) \geq 4 \\ 0 & if \deg(u) \leq 3 \end{cases}$

For $W_{1,n-1}, T_{dom^{4\alpha}}(G) = \sum_{u \in V(G)}[\deg(u)C_4] = \sum_{u \in V(G)}[(n-1)\, C_4] = (n-1)\, C_4$ (∵only one vertex of degree $(n-1)$ and $dom^{4\alpha} = 0$ for the remaining vertices of degree equal to 3)

Therefore, $CDV_4(W_{1,n-1}) = \frac{T_{dom^{4\alpha}}(G)}{nC_2} = \frac{(n-1)\, C_4}{nC_2}$

Proof of (iv) we have $T_{dom^{(n-1)\alpha}}(G) = \begin{cases} \sum_{u \in V(G)}[\deg(u)C_{n-1}] & if \deg(u) = n-1 \\ 0 & if \deg(u) < n-1 \end{cases}$

For $W_{1,n-1}, T_{dom^{(n-1)\alpha}}(G) = \sum_{u \in V(G)}[(n-1)\, C_{n-1}] = 1$

Therefore, $CDV_{n-1}(W_{1,n-1}) = \frac{T_{dom^{(n-1)\alpha}}(G)}{nC_2} = \frac{1}{nC_2} = \frac{2}{n(n-1)}$

Corollary 2.3.4.4: For Bipartite graph $K_{n,m}$

(i) If $n = m$, $T_{dom^{2\alpha}}(K_{n,m}) = \begin{cases} n^3 & if \deg(u) \geq 2 \\ 0 & if \deg(u) = 1 \end{cases}$ and

$CDV_2(K_{n,m}) = \frac{n^3}{(n+m)C_2}$

If $n < m$ or $m < n$, $T_{dom^{2\alpha}}(K_{n,m}) = \begin{cases} \frac{nm(n+m)}{2} & if \deg(u) \geq 2 \\ 0 & if \deg(u) = 1 \end{cases}$ and

$CDV_2(K_{n,m}) = \frac{nm(n+m)}{2.\ (n+m)C_2}$

(ii) If $n = m$, $T_{dom^{3\alpha}}(K_{n,m}) = \begin{cases} 2n.nC_3 & if \deg(u) \geq 3 \\ 0 & if \deg(u) \leq 2 \end{cases}$ and

$CDV_3(K_{n,m}) = \frac{2n.nC_3}{(n+m)C_2}$

If $n < m$ or $m < n$, $T_{dom^{3\alpha}}(K_{n,m}) = \begin{cases} n.mC_3 + m.nC_3 & if \deg(u) \geq 3 \\ 0 & if \deg(u) \leq 2 \end{cases}$ and

$CDV_3(K_{n,m}) = \frac{n.mC_3+m.nC_3}{(n+m)C_2}$

(iii) If $n = m$, $T_{dom^{4\alpha}}(K_{n,m}) = \begin{cases} 2n.nC_4 & if \deg(u) \geq 4 \\ 0 & if \deg(u) \leq 3 \end{cases}$ and

$CDV_4(K_{n,m}) = \frac{2n.nC_4}{(n+m)C_2}$

If $n < m$ or $m < n$, $T_{dom^{4\alpha}}(K_{n,m}) = \begin{cases} n.mC_4 + m.nC_4 & if \deg(u) \geq 4 \\ 0 & if \deg(u) \leq 3 \end{cases}$ and

$CDV_4(K_{n,m}) = \frac{n.mC_4+m.nC_4}{(n+m)C_2}$

. . . . ……….

Proof of (i) By theorem 2.1.7, $T_{dom^{2\alpha}}(G) = \begin{cases} \sum_{u \in V(G)}[\deg(u)C_2] + m & if \deg(u) \geq 2 \\ 0 & if \deg(u) = 1 \end{cases}$

For $K_{n,m}$,

If $n = m$, $T_{dom^{2\alpha}}(K_{n,m}) = \sum_{u \in V(G)}[\deg(u)C_2] + m$

$$= (n+m)(nC_2) + nm = 2n.nC_2 + n^2$$

$$= \frac{2n.\ n\,(n-1)}{2} + n^2 = n^3$$

Therefore, $CDV_2(K_{n,m}) = \frac{T_{dom^{2\alpha}}(G)}{nC_2} = \frac{n^3}{(n+m)C_2}$

If $n < m$ or $m < n$, $T_{dom^{2\alpha}}(K_{n,m}) = \sum_{u \in V(G)}[\deg(u)C_2] + m$

$$= n.mC_2 + m.nC_2 + nm = \frac{n.m(m-1)}{1.2} + \frac{m.n(n-1)}{1.2} + nm$$

$$= \frac{n.m(m-1)+m.n(n-1)+2nm}{2} = \frac{nm(n+m)}{2}$$

Therefore, $CDV_2(K_{n,m}) = \frac{T_{dom^{2\alpha}}(G)}{nC_2} = \frac{\left(\frac{nm(n+m)}{2}\right)}{((n+m)C_2)} = \frac{nm(n+m)}{2.\ ((n+m)C_2)}$

Proof of (ii) By theorem 2.2.4, For any connected graph G,

$$T_{dom^{3\alpha}}(G) = \begin{cases} \sum_{u \in V(G)}[\deg(u)C_3] & if \deg(u) \geq 3 \\ 0 & if \deg(u) \leq 2 \end{cases}$$

For $K_{n,m},\ and\ n = m, T_{dom^{3\alpha}}(G) = \sum_{u \in V(G)}[\deg(u)C_3] = \ n.mC_3 + m.nC_3$

$$= n.nC_3 + n.nC_3 = 2n.nC_3, \quad (\because n = m)$$

Therefore, $CDV_3(K_{n,m}) = \frac{T_{dom^{3\alpha}}(G)}{nC_2} = \frac{2n.nC_3}{(n+m)C_2}$

If $n < m\ or\ m < n,\ T_{dom^{3\alpha}}(K_{n,m}) = \sum_{u \in V(G)}[\deg(u)C_3]$

$$= n.mC_3 + m.nC_3$$

Therefore, $CDV_3(K_{n,m}) = \frac{T_{dom^{3\alpha}}(G)}{nC_2} = \frac{n.mC_3+m.nC_3}{((n+m)C_2)}$

Proof of (iii) By theorem 2.2.4, For any connected graph G,

$$T_{dom^{4\alpha}}(G) = \begin{cases} \sum_{u \in V(G)}[\deg(u)C_4] & if \deg(u) \geq 4 \\ 0 & if \deg(u) \leq 3 \end{cases}$$

For $K_{n,m}, T_{dom^{4\alpha}}(G) = \sum_{u \in V(G)}[\deg(u)C_4] = \ n.mC_4 + m.nC_4$

$$= n.nC_4 + n.nC_4 = 2n.nC_4, \quad (\because n = m)$$

$\therefore CDV_4(K_{n,m}) = \frac{T_{dom^{4\alpha}}(G)}{nC_2} = \frac{2n.nC_4}{(n+m)C_2}$

If $n < m\ or\ m < n,\ T_{dom^{4\alpha}}(K_{n,m}) = \sum_{u \in V(G)}[\deg(u)C_4]$

$$= n.mC_4 + m.nC_4$$

Therefore, $CDV_4(K_{n,m}) = \frac{T_{dom^{4\alpha}}(G)}{nC_2} = \frac{n.mC_4+m.nC_4}{((n+m)C_2)}$

The following corollaries are direct by the theorems 2.1.7, 2.2.4, 2.3.4

Corollary 2.3.4.5: For any regular graph G,

$$CDV_2(G) = \begin{cases} \frac{\sum_{i=1}^{n} n.[deg(v_i)C_2]+m}{nC_2} & if\ \deg(v_i) \geq 2 \\ 0 & if\ \deg(v_i) = 1 \end{cases},$$

$$CDV_3(G) = \begin{cases} \frac{\sum_{i=1}^{n} n.[deg(v_i)C_3]}{nC_2} & if\ \deg(v_i) \geq 3 \\ 0 & if\ \deg(v_i) \leq 2 \end{cases},$$

$$CDV_4(G) = \begin{cases} \frac{\sum_{i=1}^{n} n.[deg(v_i)C_4]}{nC_2} & if\ \deg(v_i) \geq 4 \\ 0 & if\ \deg(v_i) \leq 3 \end{cases},$$

. . . ….. . . . .

$$CDV_{n-1}(G) = \begin{cases} \frac{\sum_{i=1}^{n} n.[deg(v_i)C_{n-1}]}{nC_2} & if\ \deg(v_i) = n-1 \\ 0 & if\ \deg(v_i) < n-1 \end{cases}$$

Corollary 2.3.4.6: For any connected acyclic graph Tree T,

$$CDV_2(G) = \begin{cases} \frac{\sum_{i=1}^{n}[deg(v_i)C_2]+m}{nC_2} & if \ \deg(v_i) \geq 2 \\ 0 & if \ \deg(v_i) = 1 \end{cases},$$

$$CDV_3(G) = \begin{cases} \frac{\sum_{i=1}^{n}[deg(v_i)C_3]}{nC_2} & if \ \deg(v_i) \geq 3 \\ 0 & if \ \deg(v_i) \leq 2 \end{cases},$$

$$CDV_4(G) = \begin{cases} \frac{\sum_{i=1}^{n}[deg(v_i)C_4]}{nC_2} & if \ \deg(v_i) \geq 4 \\ 0 & if \ \deg(v_i) \leq 3 \end{cases}, \ldots\ldots,$$

$$CDV_{n-1}(G) = \begin{cases} \frac{\sum_{i=1}^{n}[deg(v_i)C_{n-1}]}{nC_2} & if \ if \ \deg(v_i) = n-1 \\ 0 & if \ \deg(v_i) < n-1 \end{cases}$$

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